3) An oil company discovered an oil reserve of 200 million barrels. For time \( t > 0 \), in years, the company's extraction plan is a linear declining function of time as follows:

\[ q(t) = a - bt, \]

where \( q(t) \) is the rate of extraction of oil in millions of barrels per year at time \( t \) and \( b = 0.15 \) and \( a = 16 \).

(a) How long does it take to exhaust the entire reserve?

\[ \text{time} = \text{years} \]

(b) The oil price is a constant 40 dollars per barrel, the extraction cost per barrel is a constant 18 dollars, and the market interest rate is 11 percent per year, compounded continuously. What is the present value of the company's profit?

\[ \text{value} = \text{millions of dollars} \]

\[ a = 16 \text{ Mb/yr} \quad b = 0.15 \text{ Mb/yr}^2 \quad b/a = 0.009375 \]

\[ 200 = \int_0^T (a - bt) \, dt = aT - \frac{b}{2} T^2 \]

\[ \frac{b}{2} T^2 - aT + 200 = 0 \]

\[ T = \frac{a \pm \sqrt{a^2 - 400b}}{b} = 13.3 \text{ yr} \quad (+ \text{ solution is unphysical}) \]

b) The extracted oil must be discounted by the future value value of money:

\( (e^{-rt}, \quad \text{where} \ r = 0.11) \). The total discounted quantity is then given by

\[ Q_D = \int_0^T (a - bt)e^{-rt} \, dt = a \int_0^T e^{-rt} \, dt - b \int_0^T e^{-rt} \, dt \]

\[ = \frac{a}{r} (1 - e^{-rT}) + b \left[ \frac{1}{r} \left( T + \frac{1}{r} \right) e^{-rT} + \frac{1}{r^2} \right] \]

\[ = \left[ \frac{b}{r} \left( T + \frac{1}{r} \right) - \frac{a}{r} \right] e^{-rT} + \frac{1}{r} \left( a + \frac{b}{r} \right) \]

Plugging in the given values yields a discounted quantity of 119.15 Mb. At a constant price differential of $22 / b profit, the net present value of the company is $2,621M