Let $Y \sim \text{Uniform}(\alpha,\beta)$ $\alpha,\beta \in R$ and $\alpha < \beta$. Show that

$P\{(1/y)(Y-\alpha) > \beta-Y\} = 1/(1+y)$

Solution:

$P\{(1/y)(Y-\alpha) > \beta-Y\} = P\{(Y-\alpha) > y(\beta-Y)\} = P\{Y-\alpha > y\beta-yY\}$

$= P\{Y+yY-\alpha > y\beta\} = P\{Y+yY > \alpha+ y\beta\} = P\{(1+y)Y > \alpha+ y\beta\}$

$= P\{Y > (\alpha+ y\beta)/(1+y)\} = [\beta - (\alpha+ y\beta)/(1+y)]/(\beta-\alpha) = (\beta+\beta y-\alpha-\beta y)/[(1+y)(\beta-\alpha)]$

$= (\beta-\alpha)/ [(1+y)(\beta-\alpha)] = 1/(1+y)$