1) If n = 100 and p = 0.02 in a binomial experiment, does this satisfy the rule for a normal approximation? Why or why not?

No, because np = 100(0.02) = 2. The value of np must be greater than or equal to 5 to use the normal approximation.

2) Find the z-score for the standard normal distribution where:
P(z 47.

Question is not clear. See additions at the bottom of this document.

2. Find the area under the standard normal curve:
I. to the right of z = -2.11
II. to the left of z = -2.11

I. 0.9826

II. 0.0174

(Calculated with a statistical calculator, so there is no work to show here.)

3. Assume that the population of heights of male college students is approximately normally distributed with mean m of 71 inches and standard deviation s of 3.95 inches. Show all work.
(A) Find the proportion of male college students whose height is greater than 75 inches.

$$z = \frac{(x-\mu)}{\sigma} = \frac{75-71}{3.95} = 1.0127$$

$$P(x > 75) = P(z > 1.0127) = 0.1556$$

(B) Find the proportion of male college students whose height is no more than 75 inches.

$$z = \frac{(x-\mu)}{\sigma} = \frac{75-71}{3.95} = 1.0127$$

$$P(x < 75) = P(z < 1.0127) = 0.8444$$

4. The diameters of grapefruits in a certain orchard are normally distributed with a mean of 6.70 inches and a standard deviation of 0.60 inches. Show all work.
(A) What percentage of the grapefruits in this orchard have diameters less than 7.4 inches?

$$z = \frac{(x-\mu)}{\sigma} = \frac{7.4-6.7}{0.60} = 1.1667$$

$$P(x<7.4) = P(z<1.1667) = 0.8763$$

Expressed as a percentage: 87.63%

(B) What percentage of the grapefruits in this orchard are larger than 7.15 inches?

$$z = \frac{(x-\mu)}{\sigma} = \frac{7.15-6.7}{0.60} = 0.7500$$

$$P(x>7.15) = P(z>0.7500) = 0.2266$$

Expressed as a percentage: 22.66%

5. Find the normal approximation for the binomial probability that x = 4, where n = 12 and p = 0.7. Compare this probability to the value of P(x=4) found in Table 2 of Appendix B in your textbook.

$$\mu = np = (12)(0.7) = 8.4$$

$$\sigma = \sqrt{npq} = \sqrt{(12)(0.7)(0.3)} = 1.5875$$

$$z_1 = \frac{3.5-8.4}{1.5875} = -3.0866$$

$$z_1 = \frac{4.5-8.4}{1.5875} = -2.4567$$

$$P(x=4) \approx P(z_1 < z < z_2) = P(-3.0866 < z < -2.4567)$$

$$P(x=4) \approx P(z < -2.4567) - P(z < -3.0866) = 0.007 - 0.001 = 0.006$$

The value from the binomial distribution is 0.0078. Note that, in this case, nq < 5, so the normal approximation is not considered valid for this scenario.

6. A set of data is normally distributed with a mean of 200 and standard deviation of 50.

· What would be the standard score for a score of 100?

$$z = \frac{100 - 200}{50} = -2.0$$

· What percentage of scores is between 200 and 100?

By the Empirical Rule, 95% of the scores will be within 2 standard deviations of the mean. Since the mean is 200, and a value of 100 is 2 standard deviations below that mean, the total percentage of scores between 200 and 100 will be half of 95%, or 47.5%.

· What would be the percentile rank for a score of 100?

Again, by the Empirical Rule, since 47.5% of the scores will be between 100 and the mean, the total percentage of scores that will be below 100 is 50% - 47.5% = 2.5%. The percentile rank for a score of 100 will then be 2.5.

1. If the random variable z is the standard normal score and a > 0, is it true that P(z a)? Why or why not?

Question is not clear. See additions at the end of this document.

2. Given a binomial distribution with n = 21 and p = 0.77, would the normal distribution provide a reasonable approximation? Why or why not?

No, because the value of nq is less than 5.

$q = 1 – p = 1 – 0.77 = 0.23$

$nq = (21)(0.23) = 4.83$

To use the normal approximation to the binomial distribution, both np and nq must be greater than or equal to 5, which is not the case here.

3. Find the value of z such that approximately 8.32% of the distribution lies between it and the mean.

This is equivalent to determining the value of z where the tail area is equal to $0.5 - 0.0832 = 0.4168$.

The corresponding z value is 0.2101.

(It is assumed that this question means that 8.32% of the distribution is between the mean, or $z = 0$, and $+z$, rather than 8.32% of the distribution being between $-z$ and $+z$.)

4. Find the area under the standard normal curve for the following:

(A) $P(z > 1.86)$

0.0314

(B) $P(0 < z < 1.65)$

0.4505

(C) $P(-0.41 < z < 0.26)$

$= P(z < 0.26) - P(z < -0.41) = 0.6026 - 0.3409 = 0.2617$

5. Assume that the average annual salary for a worker in the United States is $37,500 and that the annual salaries for Americans are normally distributed with a standard deviation equal to $7,000. Find the following:

(A) What percentage of Americans earn below $27,000?

$$z = \frac{27{,}000 - 37{,}500}{7{,}000} = -1.50$$

$$P(x < 27{,}000) = P(z < -1.5) = 0.0668$$

Expressed as a percentage, 6.68%

(B) What percentage of Americans earn above $45,000?

$$z = \frac{45{,}000 - 37{,}500}{7{,}000} = 1.0714$$

$$P(x > 45{,}000) = P(z < 1.0714) = 0.1420$$

Expressed as a percentage, 14.20%

6. X has a normal distribution with a mean of 80.0 and a standard deviation of 4.0. Find the following probabilities:
(A) P(x < 78.0)

$$z = \frac{78 - 80}{4} = -0.500$$

$$P(x < 78) = P(z < -0.500) = 0.3085$$

(B) P(75.0 < x 82.0)

$$z_1 = \frac{75 - 80}{4} = -1.25$$

$$z_2 = \frac{82 - 80}{4} = 0.50$$

$$P(75.0 < z < 82.0) = P(-1.25 < z < 0.50)$$
$$= P(z < 0.50) - P(z < -1.25) = 0.6915 - 0.1056 = 0.5859$$

7. Answer the following:
(A) Find the binomial probability P(x = 5), where n = 12 and p = 0.60.

$$P(x=5)=C(12,5)*(0.6)^5*(1-0.6)^{12-5}=0.1009$$

(B) Set up, without solving, the binomial probability P(x is at most 5) using probability notation.

$$P(x\le 5)=P(x=0)+P(x=0)+P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)$$

Not sure if you need the following, but here is the whole calculation set-up:

$$\begin{aligned}P(x\le 5)=&C(12,0)*(0.6)^0*(1-0.6)^{12}+C(12,1)*(0.6)^1*(1-0.6)^{11}\\&+C(12,2)*(0.6)^2*(1-0.6)^{10}+C(12,3)*(0.6)^3*(1-0.6)^9\\&+C(12,4)*(0.6)^4*(1-0.6)^8+C(12,5)*(0.6)^5*(1-0.6)^7\end{aligned}$$

(C) How would you find the normal approximation to the binomial probability P(x = 5) in part A? Please show how you would calculate μ and σ in the formula for the normal approximation to the binomial, and show the final formula you would use without going through all the calculations.

$\mu = np = (12)(0.6) = 7.2$

$$\sigma=\sqrt{npq}=\sqrt{(12)(0.6)(0.4)}=1.6971$$

$$z_1=\frac{4.5-7.2}{1.6971}$$

$$z_2=\frac{5.5-7.2}{1.6971}$$

$$P(x=5)\approx P(z_1<z<z_2)=P(z<z_1)-P(z<z_2)$$

2) Find the *z*-score for the standard normal distribution where:
P(z< -a) = 0.2451

A left tail area of 0.2451 corresponds to a z value of -0.69, so a = 0.69

1. A set of 50 data values has a mean of 40 and a variance of 25.

I. Find the standard score (z) for a data value = 47.

Since the variance is 25, the standard deviation is √25, which equals 5.

$$z = \frac{47 - 40}{5} = 1.40$$

II. Find the probability of a data value > 47.

$$P(x > 47) = P(z > 1.4) = 0.0808$$