In the United States, voters who are neither Democrats or Republicans are called Independents. It is believed that 10% of all voters are Independents. A survey asked 25 people to identify themselves as Democrat, Republican or Independent.

A. What is the probability that none of the people are Independent?

B. What is the probability that fewer than five are Independent?

C. What is the probability that more than two are Independent?

Solution:

The solutions for these questions use the binomial probability calculation:

\[ P(x, n) = \binom{n}{x} \cdot p^x \cdot q^{n-x} \]

where \( \binom{n}{x} = \frac{n!}{x!(n-x)!} \)

A. What is the probability that none of the people are Independent?

For this problem, \( n = 25 \), \( x = 0 \), \( p = 0.10 \), and \( q = 0.90 \)

\[ P(0, 25) = _{25}C_0 \cdot (0.10)^0 \cdot (0.90)^{25-0} = \frac{25!}{0!(25-0)!} \cdot (0.10)^0 \cdot (0.90)^{25} = 0.0718 \]

B. What is the probability that fewer than five are Independent?

The probability that fewer than five are Independent is equal to the sum of the probabilities that 0, 1, 2, 3, or 4 are independent:


\[ P(0, 25) = _{25}C_0 \cdot (0.10)^0 \cdot (0.90)^{25-0} = \frac{25!}{0!(25-0)!} \cdot (0.10)^0 \cdot (0.90)^{25} = 0.0718 \]

\[ P(1, 25) = _{25}C_1 \cdot (0.10)^1 \cdot (0.90)^{25-1} = \frac{25!}{1!(25-1)!} \cdot (0.10)^1 \cdot (0.90)^{24} = 0.1994 \]

\[ P(2, 25) = _{25}C_2 \cdot (0.10)^2 \cdot (0.90)^{25-2} = \frac{25!}{2!(25-2)!} \cdot (0.10)^2 \cdot (0.90)^{23} = 0.2659 \]

\[ P(3, 25) = _{25}C_3 \cdot (0.10)^3 \cdot (0.90)^{25-3} = \frac{25!}{3!(25-3)!} \cdot (0.10)^3 \cdot (0.90)^{22} = 0.2265 \]

\[ P(4, 25) = _{25}C_4 \cdot (0.10)^4 \cdot (0.90)^{25-4} = \frac{25!}{4!(25-4)!} \cdot (0.10)^4 \cdot (0.90)^{21} = 0.1384 \]

Then \( P(x \leq 5) = 0.0718 + 0.1994 + 0.2659 + 0.2265 + 0.1384 = 0.9020 \)
C. What is the probability that more than two are Independent?

The probability that more than two are Independent is equal to one, minus the probabilities that 0, 1, or 2 are Independent.

\[ P(x > 2, 25) = 1 - P(0, 25) - P(1, 25) - P(2, 25) \]

Using the values calculated in part B:

\[ P(x > 2, 25) = 1 - 0.0718 - 0.1994 - 0.2659 = 0.4629 \]