

5. Consider the curve $y = x^2 - 3x + 2$.

(a) Sketch the curve over the range $x = 0$ to $x = 3$.

(b) Evaluate the area bounded by the curve and the x-axis between the limits $x=0$ and $x=1$.

a)

The graph of this quadratic equation will be a parabola. Since the coefficient of the x^2 term is positive, the parabola will open toward the top.

The solutions to $x^2 - 3x + 2 = 0$ are:

$$\begin{aligned}x^2 - 3x + 2 &= 0 \\(x - 2)(x - 1) &= 0 \\x - 2 = 0 &\quad x - 1 = 0 \\x = 2 &\quad x = 1\end{aligned}$$

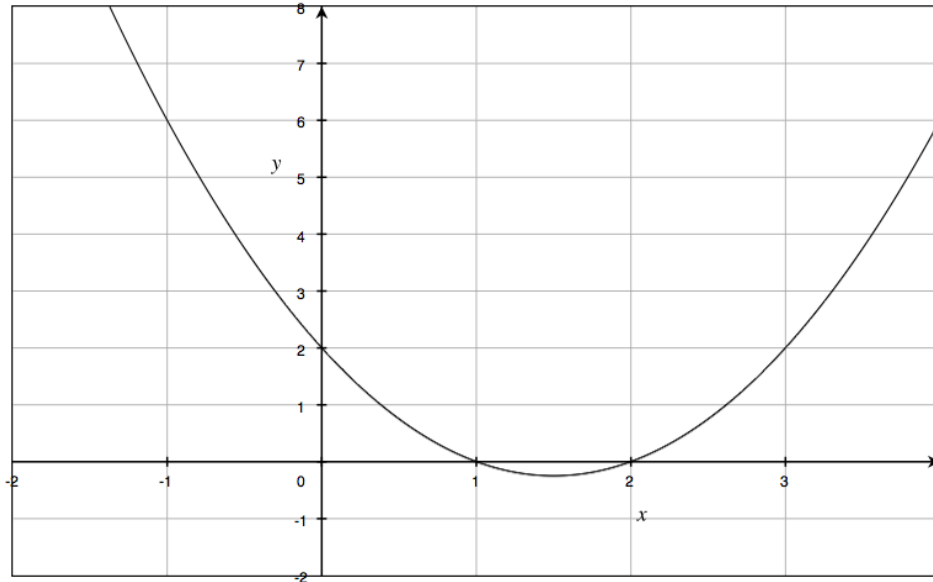
So the x-intercepts are $(2, 0)$ and $(1, 0)$.

The x-coordinate of the vertex of the parabola is $x = -b/2a = -(-3) / 2 = 3/2$.

The y-coordinate of the vertex is then $(3/2)^2 - 3(3/2) + 2 = -1/4$

The vertex is located at $(3/2, -1/4)$

The curve looks like this:



(b) Evaluate the area bounded by the curve and the x-axis between the limits $x=0$ and $x=1$.

$$\int_0^1 x^2 - 3x + 2 \, dx = \left[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x \right]_0^1$$

$$= \frac{1}{3}(1)^3 - \frac{3}{2}(1)^2 + 2(1)$$

$$= \frac{1}{3} - \frac{3}{2} + 2$$

$$= \frac{5}{6}$$